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HEATED WING ANALYSIS METHOD - FORMULAS AND DETAILS

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BY

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STRUCTURES SECTION

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SUMMARY

A method for analyzing heated thin wings was developed under a Grumman Advanced Development Program and is reported in the paper, "A Matrix Force Method for Analyzing Heated Wings, Including Large Deflections".<sup>(1)</sup> This paper was presented at the symposium "Structural Dynamics of High Speed Flight", sponsored by the Aerospace Industries Association and The Office of Naval Research in Los Angeles, California, April 24-26, 1961.

The present report contains some details on certain aspects of the method, which were of necessity omitted from the paper. Several examples illustrating the application of the method are contained in the paper and might be referred to while this report is being read.

This report and Refs. 2, 3 and 4 document the Grumman matrix force method of redundant structure analysis as developed to date. The same notation is used throughout these references.

SYMBOLS

$A, P, k, q, \sigma$	defined in Ref. (2)
$Z, Z'$	lateral loads induced in deflected capstrips and shear panels respectively
$l$	length of a spar or rib segment between panel points
$P'$	kick load
$B, C$	constants arising in systematic calculation of kick loads
$D, F$	matrices arising in calculation of heated structure flexibilities
$\bar{q}$	buckling load parameter
$L, L'$	geometry dependent constants relating the displacements at the member loads to strains associated with the member loads

## Subscripts:

$i, j, m, n$	defined in Ref. (2)
$0, 1, 2$ or $A, B, C, D$	particular designations corresponding to the general subscripts $i, j, m, n$ (used in the derivations to identify particular member loads or panel points)
$J$	pertaining to the $j$ th member load of the structure
$a$	pertaining to the $a^{\text{th}}$ member of the structure
$\theta$	pertaining to the $\theta^{\text{th}}$ thermal condition

## THEORY

### A. Introduction

The basic formulation for the Grumman matrix force method of redundant structure analysis was developed in Ref. 2. The method was expanded later in company reports to include some effects that are peculiar to certain types of structures or applications. Specifically, Ref. 3 outlines the procedure for calculation of stresses and deflections resulting from non-uniform thermal strain, while Ref. 4 explains the inclusion of Poisson's ratio and sweep coupling effects.

A recent paper, Ref. 1, further extended the method, as applied to the particular case of a thin wing, to include (1) the influence of in-plane thermal stresses on the flexibility of the wing for lateral load, (2) the limiting case of increased flexibility from this cause, namely thermal buckling, and (3) the stiffening effect that results from large deflections. The emphasis in the paper was on the basic formulation and examples, and due to space limitations, only the most important formulas could be given in some instances.

The following sections give a more detailed development of the formulas.

### B. Linear Analysis

It will be assumed in the discussion that follows that the wing structure is symmetrical about a mean chord plane and that it can be idealized in the stringer, shear panel and shear web manner. This is one of the standard methods for idealizing a multi-spars, multi-rib wing and consists of representing the wing by capstrips carrying axial load only, interconnected by shear panels which can carry tangential edge loads only. Using this approach, one lumps the covers into equivalent capstrips over the spars and ribs; the contributions of the spars and ribs themselves are calculated and added in. If appropriate, intermediate shear lag members are introduced as well. Because of the shear panel assumption, the axial loads in the capstrips vary linearly from one panel point to the next.

Based upon this idealization, the member loads in the statically determinate structure and the member flexibilities can be determined by the procedures already given in Refs. 2, 3 and 4. Following these procedures further, one can determine the stress distribution and the deflections due to applied loads and/or temperature changes under the assumptions of linear behavior of the structure.

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The remaining sections are concerned with the procedures which are applicable in cases where in-plane and out-of-plane components of load and deflection cannot be directly superposed.

### C. Interaction Problem - Small Deflections

The problem of analyzing a wing under an arbitrary temperature distribution, neglecting the interaction effect, can be handled by considering the wing to be subjected to two component temperature distributions, symmetrical and anti-symmetrical about the mean chord plane respectively, which when superposed, add up to the given distribution. The wing is analyzed under each separately, and then the resulting stresses are added algebraically. The temperature distribution symmetrical about the mean plane will give rise to stresses symmetrical about the mean plane and displacements in the plane, while the anti-symmetrical temperature distribution will produce anti-symmetrical stresses and displacements normal to the plane. The latter add directly to the stresses and deflections caused by lateral applied loads.

An obvious assumption in the foregoing procedure, and a valid one in many cases, is that the two analyses can be treated independently. However, if the in-plane stresses are large enough, there can be noticeable interaction between the two thermal stress distributions themselves and between the in-plane stresses and the lateral displacements due to applied loads. In this case the flexibility influence coefficient matrix for the structure may be appreciably altered.

In the development of the method for accounting for this effect it is assumed that the deflections are small in the sense that no membrane type stresses are developed due to deflections. The two idealized structures used in the symmetrical and anti-symmetrical analyses for a given case will be geometrically identical. Element flexibilities however may be different, inasmuch as the stresses in one case depend upon the bending and torsional flexibility and in the other upon the in-plane flexibility. With the "cold" structure flexibility influence coefficient matrix and the symmetrical and anti-symmetrical thermal stress distributions already calculated by the methods given in Refs. 2 and 3, the two analyses can be coupled by the procedure to follow.

In the development of the coupling formulas it is convenient to consider first the symmetrical thermal stress distribution; in this case the loads in pairs of upper and lower capstrips and shear panels are identical. This load distribution is self equilibrating only as long as the structure remains undeflected laterally.

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When the spars and ribs bend and the shear panels become warped, resultant forces normal to the plane are induced. To illustrate, consider the segment of spar or rib shown schematically in Fig. 1. As shown, the average slope between panel points is taken as the element slope. To calculate the magnitude of the lateral force, which will be called a kick load and denoted by  $Z$ , consider vertical components of forces at the panel point. Summation of these forces gives

$$Z = 2q_1 \left[ \frac{(\delta_0 - \delta_1)}{l_1} - \frac{(\delta_1 - \delta_2)}{l_2} \right] \quad (1)$$

where  $2q_1$  is the total load in the upper and lower capstrips;  $\delta_0$ ,  $\delta_1$ , and  $\delta_2$  are the deflections at three consecutive panel points, and  $l_1$  and  $l_2$  are the distances between them.

Fig. 2 shows a pair of deflected quadrilateral shear panels, together with their projection on the undeflected mean chord plane. The mean positions of the four corners are identified by the four panel point deflections  $\delta_A$ ,  $\delta_B$ ,  $\delta_C$ , and  $\delta_D$ . The in-plane shear flows  $k_{AB}q_1$ , etc. have a lateral resultant which is called  $Z'$  and can be calculated by again summing forces vertically. Thus,

$$Z' = 2q_1 \left[ (k_{AB} + k_{AD})\delta_A - (k_{AB} + k_{BC})\delta_B + (k_{BC} + k_{CD})\delta_C - (k_{AD} + k_{CD})\delta_D \right] \quad (2)$$

In the case of the shear panels, the force in eqn. (2), which is assumed to act at the intersection of the diagonals, is replaced by a statically equivalent set of forces at the four corners of the panel.

In the formulas to follow, the matrix notation of Ref. 2 will be extended and used. The kick loads will be written in column matrix arrangement and designated  $\{P'_m\}$ . They are dependent upon the deflections, which in column order are called  $\{\delta_n\}$ , and the in-plane member loads which, for convenience, are written in diagonal matrix form. The latter are designated  $[q_{ij}]$ . The formula from which the kick loads are obtained is then

$$\{P'_m\} = [F_{mn}] \{\delta_n\} \quad (3)$$

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where

$$[F_{mn}] = [B_{mi}] [q_{ia}] [C_{in}] \quad (4)$$

The  $[B_{mi}]$  and  $[C_{in}]$  matrices are made up of geometry dependent constants derived from eqns. (1) and (2). They can be obtained by a systematic procedure which is discussed later.

Recalling from Ref. 3 that the deflections due to lateral loads and anti-symmetrical thermal strains are

$$[A_{mn}] \{P_n\} + \{\delta_{me}\},$$

the total deflections may be written

$$\{\delta_n\} = [A_{mn}] \{P_n\} + \{\delta_{me}\} + [A_{mn}] [F_{mn}] \{\delta_n\} \quad (5)$$

Solving for  $\{\delta_n\}$ ,

$$\{\delta_n\} = [D_{nn}]^{-1} [A_{nn}] \{P_n\} + [D_{nn}]^{-1} \{\delta_{me}\} \quad (6)$$

where

$$[D_{nn}] = [I] - [A_{nn}] [F_{nn}]$$

It is seen that  $[D_{nn}]^{-1} [A_{nn}]$  is the required matrix of flexibility influence coefficients. Multiplication of  $[A_{nn}]$  by  $[D_{nn}]^{-1}$  introduces the change in flexibility due to in-plane thermal stresses, while multiplication of  $\{\delta_{me}\}$  by  $[D_{nn}]^{-1}$  takes care of the interaction between the symmetrical thermal stresses and the thermal displacements discussed previously.

With the  $\{\delta_n\}$  known, it is now possible to substitute in eqn. (3) to determine the kick loads  $\{P'_m\}$ . With these known, the total anti-symmetrical components of the member loads may now be determined as

$$\{q_{ia}\}_{as} = \frac{1}{2} [F_{im}] \{P_m\} + \frac{1}{2} \{q_{ia}\}_{as} + \frac{1}{2} [F_{im}] \{P'_m\} \quad (7)$$



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In eqn. (5), the subscript "as" refers to the lateral, or anti-symmetrical, analysis. The + and - signs indicate the top and bottom surface member loads respectively. In those cases where the section properties are the same for the anti-symmetrical and the symmetrical analyses, the in-plane loads,  $\{q_o\}$  may be added directly to the loads given by eqn. (7). If this is not the case, then it is necessary to calculate actual stresses for the two cases and then superpose.

#### D. Analysis - Systematic Calculation of $B_{mi}$ and $C_{in}$

In order that the method be appropriate for routine use, the entire set of kick loads  $\{P'_m\}$  must be obtainable by a systematic procedure for any given in-plane thermal stress distribution. The result of one such method is indicated in eqn. (4).

Consider first a typical segment of spar or rib, as illustrated in Fig. (3). In-plane member loads  $q_A$  and  $q_B$  are assumed to act on this particular pair of capstrips; deflections  $\delta_C$  and  $\delta_D$ , together with the corresponding kick loads  $P'_C$  and  $P'_D$  are shown as well. Referring now to eqn. (1), the contributions of this pair of capstrips to  $P'_C$  and  $P'_D$  are

$$P'_C = 2q_A \frac{\delta_D - \delta_C}{\lambda_A}, \quad P'_D = 2q_B \frac{\delta_C - \delta_D}{\lambda_A}$$

These expressions can be written in matrix form:

$$\begin{bmatrix} P'_C \\ P'_D \end{bmatrix} = \begin{bmatrix} 2 & \\ & 2 \end{bmatrix} \begin{bmatrix} q_A \\ q_B \end{bmatrix} \begin{bmatrix} -1/\lambda_A & 1/\lambda_A \\ 1/\lambda_A & -1/\lambda_A \end{bmatrix} \begin{bmatrix} \delta_C \\ \delta_D \end{bmatrix}$$

The contributions of this pair of capstrips to the  $B_{mi}$  and  $C_{in}$  matrices are thus:

$$[B_{ami}] = \begin{array}{|c|c|c|} \hline & A & B \\ \hline C & 2 & \\ \hline D & & 2 \\ \hline \end{array}, \quad [C_{ain}] = \begin{array}{|c|c|c|} \hline & C & D \\ \hline A & -1/\lambda_A & 1/\lambda_A \\ \hline B & 1/\lambda_A & -1/\lambda_A \\ \hline \end{array}$$

Referring now to the typical cover shear panel shown in Fig. (4), the in-plane member load  $q_1$ , in combination with the lateral deflections of the four corners  $\delta_A, \delta_B, \delta_C$  and  $\delta_D$ , induces a kick load  $Z'$ , as described by eqn. (2). Distributing half of this load to each pair of diagonally opposite corners,

$$\begin{aligned} P'_A &= \frac{\lambda_C}{2(\lambda_A + \lambda_C)} Z' & P'_C &= \frac{\lambda_A}{2(\lambda_A + \lambda_C)} Z' \\ P'_B &= \frac{\lambda_D}{2(\lambda_D + \lambda_B)} Z' & P'_D &= \frac{\lambda_B}{2(\lambda_C + \lambda_D)} Z' \end{aligned}$$

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Substituting for  $Z'$  from eqn. (2) and rewriting in matrix form,

$$\begin{bmatrix} P'_A \\ P'_B \\ P'_C \\ P'_D \end{bmatrix} = \begin{bmatrix} l_c/(l_A + l_c) \\ l_d/(l_D + l_B) \\ l_A/(l_A + l_c) \\ l_c/(l_c + l_D) \end{bmatrix} [q_i] \begin{bmatrix} (k_{AB} + k_{AD}), -(k_{AB} + k_{EC}), (k_{EC} + k_{CD}), -(k_{AD} + k_{CD}) \end{bmatrix} \begin{bmatrix} \delta_A \\ \delta_B \\ \delta_C \\ \delta_D \end{bmatrix}$$

The contributions of this pair of cover panels to the  $B_{mi}$  and  $C_{in}$  matrices are thus:

$$[B_{ami}] = \begin{array}{|c|c|} \hline & i \\ \hline A & l_c/(l_A + l_c) \\ B & l_d/(l_D + l_B) \\ C & l_A/(l_A + l_c) \\ D & l_c/(l_c + l_D) \\ \hline \end{array}, [C_{ain}] = \begin{array}{|c|c|c|c|} \hline & A & B & C & D \\ \hline i & (k_{AB} + k_{AD}) & -(k_{AB} + k_{EC}) & (k_{EC} + k_{CD}) & -(k_{AD} + k_{CD}) \\ \hline \end{array}$$

The  $B_{ami}$  and  $C_{ain}$  matrices can easily be formed in this manner for every pair of capstrips and for every pair of cover shear panels in the structure. They may then be added together in the same way that member flexibilities are added to form the member flexibility matrix  $[\alpha_{ij}]$  of Ref. 1, to yield  $[B_{mi}]$  and  $[C_{in}]$ .

It is recommended that in numbering the member loads the numerical subscripts designating member loads acting upon any one member be reserved uniquely for that member. (For example, even though axial loads in adjacent segments of capstrips must necessarily be equal where they join together, it is recommended that the member load numerical subscripts be selected such that they do not repeat.) This is desirable for several reasons. In this instance, it is beneficial because when the  $B_{ami}$  and  $C_{ain}$  matrices for all of the members are collected into the  $B_{mi}$  and  $C_{in}$  matrices, there will be no overlapping of individual elements. This in turn means less chance for error.

#### E. Thermal Buckling

It is well known that the increased flexibility of structures for lateral load is related to the proximity of the in-plane load to buckling. The same matrices used to calculate the heated structure flexibilities can be used to determine the factor by which a given in-plane loading distribution must be increased to cause buckling.

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If the structure is subjected to in-plane loads only, then according to eqn. (5),

$$\{\delta_n\} = [A_{mn}] [F_{mn}] \{\delta_n\} \quad (8)$$

Examination of eqn. (8) indicates that equilibrium configurations other than the undeflected case may be possible. If they are, then the deflections  $\delta_n$  and the loads  $q_{in}$  entering  $[F_{mn}]$  must satisfy eqn. (8) and can be determined from it. Assuming that the in-plane loads increase proportionally, then a given load distribution becomes at buckling  $\bar{q}(q_{in})$ .  $\bar{q}$  is the factor which when multiplied by each of the in-plane loads gives the load distribution causing buckling. Eqn. (8) can now be written as

$$\frac{1}{\bar{q}} \{\delta_n\} = [A_{mn}] [F_{mn}] \{\delta_n\} \quad (9)$$

which is recognized as an eigenvalue equation. The lowest value  $\bar{q}$  satisfying this equation defines the in-plane loading condition for buckling. The corresponding eigenvector  $\{\delta_n\}$  gives the buckled shape.

#### F. Interaction Problem - Large Deflections

An implied assumption in linear wing analysis methods is that the lateral deflections are small enough so that the attendant straining of the middle plane is negligible. It is quite possible that this assumption may not be valid for the case of a thin, low aspect ratio wing. On the contrary, as a consequence of large deflections, there may be a stiffening of the wing due to the development of in-plane membrane type stresses which help carry the load.

In Ref. 1 a method is developed for including this effect. It permits the determination of the deflected shape for a given set of applied loads, together with the attendant stress distribution, and the flexibility of the structure for small displacements about the deflected shape. The latter is stated by means of a tangent flexibility matrix which, of course, is dependent upon the particular loading condition.

The first step in the procedure is to calculate the stress distribution in the middle plane consistent with an assumed or calculated approximate set of deflections. This calculation is similar to the calculation of in-plane stresses due to non-uniform thermal expansion; in the present case the strains are due instead to deflection.

### THEORY

Formulas for the strains in the statically determinate, stress free, deflected middle plane are given in Ref. 1 for spars and ribs and for rectangular shear panels. To complete the collection of formulas that may be needed for swept wing analyses, expressions are derived in the Appendix for shear panels of general quadrilateral shape.

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FIGURES

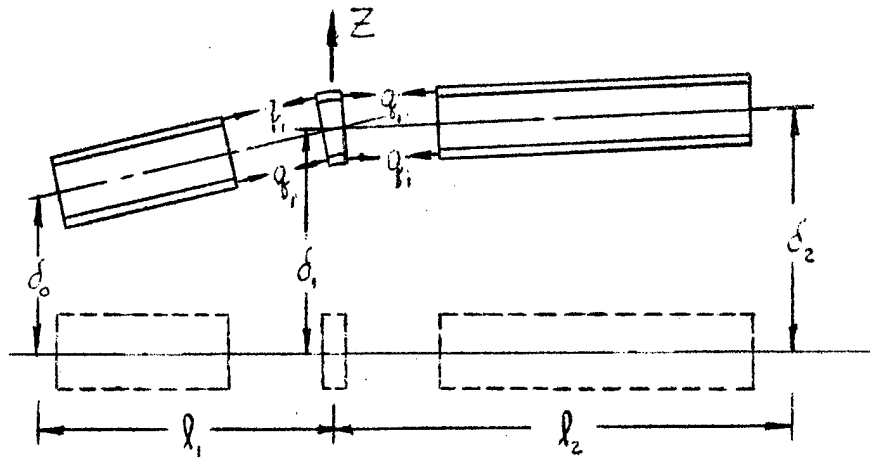


FIGURE 1.

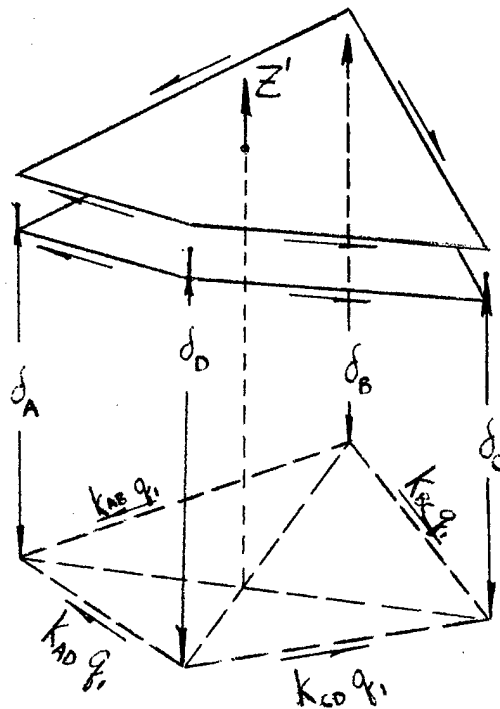


FIGURE 2.

FIGURES

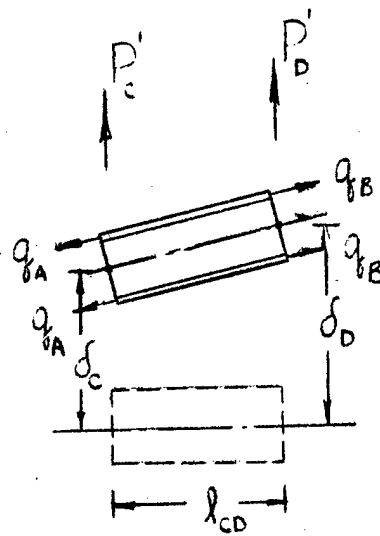


FIGURE 3.

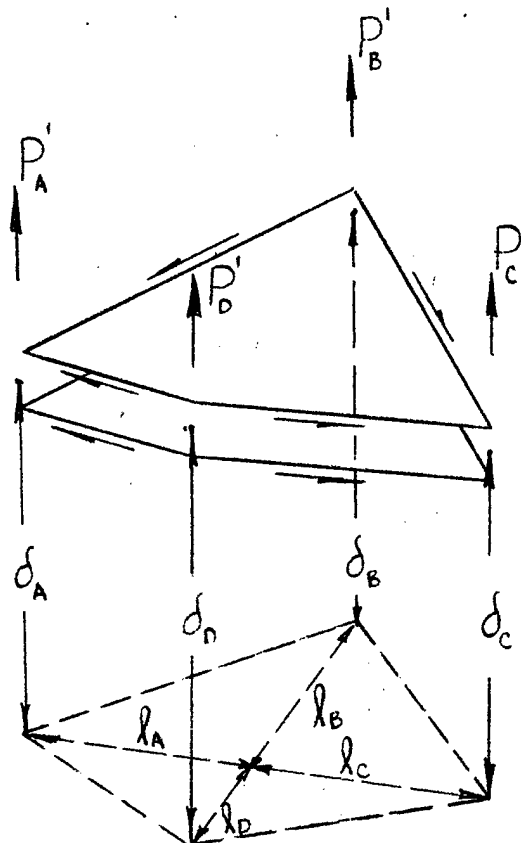


FIGURE 4.

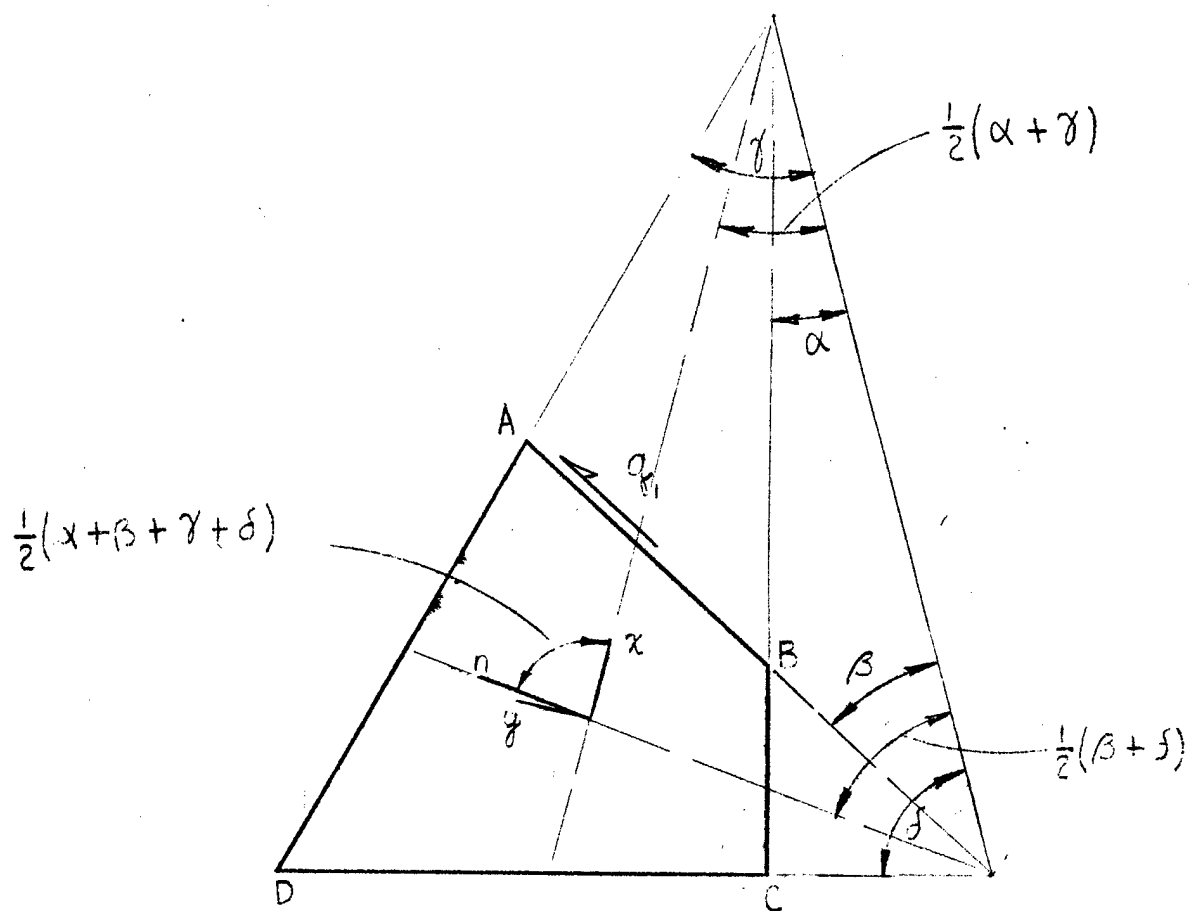


FIGURE 5.

APPENDIXMEMBRANE STRAINS IN QUADRILATERAL SHEAR PANELS1. Introduction

Ref. 3 develops the following expression for the virtual work done in a quadrilateral shear panel:

$$\int_V \tau_{sr} (2 \epsilon_x \tan \lambda + \gamma_{xy}) dV = q_b (L_{11} \epsilon_x + L'_{11} \gamma_{xy}) \quad (1A)$$

The various terms of this expression are explained in Ref. 3; in particular  $q_b$  is the member load shear flow,  $L_{11}$  and  $L'_{11}$  are functions of the shape of the panel, and  $\epsilon_x$  and  $\gamma_{xy}$  are the direct and shear strains associated with a line bisecting the angle between the two opposite sides of the quadrilateral that are most nearly parallel. See Fig. 5. It is the primary purpose of this section to derive equations for the strains  $\epsilon_x$  and  $\gamma_{xy}$  when they arise due to large deflections.

2. Derivation of Expressions for  $\epsilon_x$  and  $\gamma_{xy}$ 

Referring to Fig. 5, the bisector of the angle  $(\gamma - \alpha)$  is designated the x axis; the bisector of the angle  $(\delta - \beta)$  is called the n direction. Then in terms of the lateral deflections of the four corners  $\delta_A, \delta_B, \delta_C$  and  $\delta_D$ , the average slopes  $\frac{\partial w}{\partial x}$  and  $\frac{\partial w}{\partial n}$  are:

$$\frac{\partial w}{\partial x} = \frac{1}{2} \left[ \frac{\delta_A - \delta_D}{l_{AD}} + \frac{\delta_B - \delta_C}{l_{BC}} \right] \quad (2A)$$

$$\frac{\partial w}{\partial n} = \frac{1}{2} \left[ \frac{\delta_A - \delta_B}{l_{AB}} + \frac{\delta_D - \delta_C}{l_{CD}} \right] \quad (3A)$$

The quantities  $l_{AB}, l_{BC}, l_{CD}$  and  $l_{AD}$  are the lengths of the four sides of the quadrilateral. Use of eqn. (a), page 34 of Ref. 5 now yields for the slope in the y direction:

$$\begin{aligned} \frac{\partial w}{\partial y} = & -\frac{\partial w}{\partial x} \cot \frac{1}{2}(\alpha + \beta + \gamma + \delta) \\ & + \frac{\partial w}{\partial n} \csc \frac{1}{2}(\alpha + \beta + \gamma + \delta) \end{aligned}$$



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The strains  $\epsilon_x$  and  $\gamma_{xy}$  due to large deflections, whose finite difference equivalents for rectangular panels are given on pages 15 and 16 of Ref. 1, become

$$\epsilon_x = -\left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]$$

$$\gamma_{xy} = -\left[-\left(\frac{\partial w}{\partial x}\right)^2 \cot \frac{1}{2}(\alpha + \beta + \gamma + \delta) + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \csc \frac{1}{2}(\alpha + \beta + \gamma + \delta)\right]$$

Substituting eqns. (2A) and (3A) in the preceding gives the desired equations.

### 3. Equivalent Strain $\gamma_{eq}$

The right hand side of the virtual work expression, eqn. (1A), consists of two terms, one in  $\epsilon_x$  and the other in  $\gamma_{xy}$ . Both strains are functions of the same panel displacements  $\delta_A, \delta_B, \delta_C, \delta_D$  however, and it is therefore convenient to combine everything into a single term. This can be done in the following way. An "equivalent" shear strain  $\gamma_{eq}$  is defined such that

$$q_i L_{ii} \gamma_{eq} = q_i (L_{ii} \epsilon_x + L_{ii} \gamma_{xy})$$

Thus

$$\gamma_{eq} = \frac{L_{ii}}{L_{ii}} \epsilon_x + \gamma_{xy} \quad (4A)$$

$\gamma_{eq}$  is seen to be a quadratic function of  $\delta_A, \delta_B, \delta_C, \delta_D$ , as follows:

$$\gamma_{eq} = -[\delta_A \delta_B \delta_C \delta_D] \begin{bmatrix} a_{AA} & a_{AB} & a_{AC} & a_{AD} \\ a_{BA} & & & \\ \vdots & & & \end{bmatrix} \{\delta_A \delta_B \delta_C \delta_D\} \quad (5A)$$

where

$$a = \frac{1}{8} \frac{L_{ii}}{L_{ii}} = \frac{1}{4} \cot \frac{1}{2}(\alpha + \beta + \gamma + \delta)$$

$$b = \frac{1}{4} \csc \frac{1}{2}(\alpha + \beta + \gamma + \delta)$$

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and

$$a_{AA} = \frac{a}{l_{AD}^2} + \frac{b}{l_{AB} l_{AD}} ; \quad a_{EB} = \frac{a}{l_{BC}^2} - \frac{b}{l_{AB} l_{BC}}$$

$$a_{CC} = \frac{a}{l_{BC}^2} + \frac{b}{l_{BC} l_{CD}} ; \quad a_{DD} = \frac{a}{l_{AD}^2} - \frac{b}{l_{AD} l_{CD}}$$

$$a_{AB} = a_{BA} = \frac{a}{l_{AD} l_{BC}} + \frac{b}{2} \left( -\frac{1}{l_{AB} l_{AD}} + \frac{1}{l_{AB} l_{BC}} \right)$$

$$a_{AC} = \frac{-a}{l_{AD} l_{BC}} - \frac{b}{2} \left( \frac{1}{l_{AD} l_{CD}} + \frac{1}{l_{AB} l_{BC}} \right)$$

$$a_{AD} = \frac{-a}{l_{AD}^2} + \frac{b}{2} \left( \frac{1}{l_{AD} l_{CD}} - \frac{1}{l_{AB} l_{AD}} \right)$$

$$a_{BC} = \frac{-a}{l_{BC}^2} + \frac{b}{2} \left( -\frac{1}{l_{BC} l_{CD}} + \frac{1}{l_{AB} l_{BC}} \right)$$

$$a_{BD} = \frac{-a}{l_{AD} l_{BC}} + \frac{b}{2} \left( \frac{1}{l_{BC} l_{CD}} + \frac{1}{l_{AB} l_{AD}} \right)$$

$$a_{CD} = \frac{a}{l_{AD} l_{BC}} + \frac{b}{2} \left( \frac{1}{l_{BC} l_{CD}} + \frac{1}{l_{AD} l_{CD}} \right)$$

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